

# Mechanics of disordered auxetic metamaterials

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Auxetic materials are of great engineering interest not only because of their fascinating negative Poisson's ratio, but also due to their increased toughness and indentation resistance. These materials are typically synthesized polyester foams with a very heterogeneous structure, but the role of disorder in auxetic behavior is not fully understood. Here, we provide a systematic theoretical and experimental investigation in to the effect of disorder on the mechanical properties of a paradigmatic auxetic lattice with a re-entrant hexagonal geometry. We show that disorder has a marginal effect on the Poisson's ratio unless the lattice topology is altered, and in all cases examined the disorder preserves the auxetic characteristics. Depending on the direction of loading applied to these disordered auxetic lattices, either brittle or ductile failure is observed. It is found that brittle failure is associated with a disorder-dependent tensile strength, whereas in ductile failure disorder does not affect strength. Our work thus provides general guidelines to optimize elasticity and strength of disordered auxetic metamaterials.

## I. INTRODUCTION

Auxetic materials display the counter-intuitive property of expanding orthogonally to the stretching direction, which makes them suitable for a variety of applications [1]. General mechanisms underlying this peculiar behavior have been uncovered in the design of ordered metamaterial lattices [2–5], but their applicability to commercially available bulk auxetic foams, which are typically strongly disordered [6], remains unclear. The interest in auxetic metamaterials stems from their improved mechanical properties, such as toughness and indentation resistance [7]. Previous studies of the effect of disorder yielded a somewhat contradictory picture: Simple disordered structures designed to imitate bulk auxetics [8] include perforated sheets [9] and arrays of rigid rotating units [10–12]. In the latter case, disorder has little affect on the Poisson's ratio. In contrast, studies on disordered versions of the re-entrant hexagonal lattice [13, 14] demonstrate that disorder can change the impact resistance [15] and decrease the magnitude of the Poisson's ratio and yield strength significantly [16, 17].

In addition to the elastic properties, the fracture properties of auxetics are of interest because of their increased fracture toughness relative to conventional materials [7, 18]. It has been shown computationally that the ordered re-entrant hexagonal lattice exhibits improved fracture properties over an equivalent conventional hexagonal lattice [19], however little work exists on the failure of this lattice experimentally [20]. The presence of disorder has not been considered, even though normally disorder has a key role in fracture strength and toughness [21].

Here we study the elastic properties as well as the fracture mechanics of perfect (zero disorder) and disordered auxetic lattices. We introduce a methodology for adding disorder including the possibility to alter the topology of the lattice, which we find to be crucial for the auxetic behavior. In particular, the Poisson's ratio is shown to depend very strongly on the presence of topological defects. This is remarkable in view of the increasing interest in the role of topology in mechanical metamaterials [22–24].

## II. METHODS

### A. Lattice construction

To add disorder to the auxetic re-entrant lattice [25–27] we begin by constructing a disordered hexagonal lattice, and convert it to a re-entrant hexagonal lattice using the process shown in Fig. 1(a). The parameter  $d$  controls the amount of disorder in the initial triangular lattice, which determines the structure of the disordered re-entrant lattice. All cells in the ordered lattice are hexagons, though variations in the topology are possible in the disordered geometries. Fig. 1(b) shows examples of disordered lattices with different values of  $d$ , and demonstrates that the topology of the lattice is also controlled by this parameter, with only very disordered samples including topological defects.

### B. 3D printing

Experimentally, samples are prepared for tension [Fig. 1(c)] and compression [Fig. 1(d)] through 3D printing. The force is applied in one of two orientations, labelled

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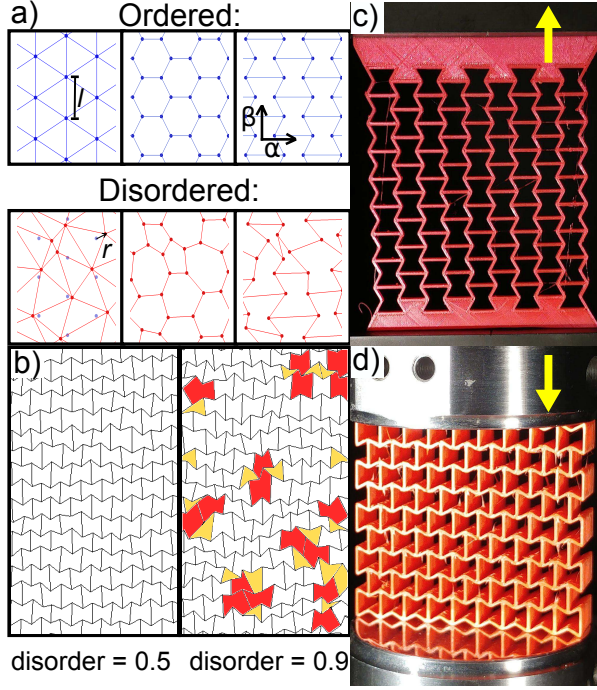


Figure 1. a) **Construction of re-entrant lattices.** The top row shows a triangular lattice with edge length  $l$  (left), its hexagonal dual lattice (middle), and the re-entrant lattice (right). The bottom row shows the disordered triangular lattice (left), where each vertex has been shifted from its ordered position by  $\mathbf{r} = [ld_x\sqrt{3}/4, ld_y/4]$ , where  $d_x$  and  $d_y$  are uniformly distributed random numbers between  $-d$  and  $d$ . Its dual lattice (middle) is converted to the disordered re-entrant lattice (right) by moving each vertex such that its most horizontal bond is lengthened by 50%. b) **Effect of disorder.** Examples of re-entrant lattices with different values of the disorder. Topological defects are coloured in red (heptagons) and yellow (pentagons). c) **Experimental tension setup.** d) **Experimental compression setup.** Force is applied in the direction of the yellow arrow.

as  $\alpha$  and  $\beta$  in Fig. 1(a). The 3D models of each geometry have been produced with a bond width of 0.4 mm. Samples are manufactured by means of 3D printing, fused deposition modeling technique (FDM). In this method structures are produced by laying down many successive thin layers of molten plastic. Each layer thickness is 0.4 mm. For each experiment the lattice dimensions (in cm) are: Fig. 2(a) and 2(c) =  $7.5 \times 9 \times 0.15$ , Fig. 2(b) =  $4.5 \times 4.5 \times 1.0$ , Fig. 4(a)  $\alpha = 9.5 \times 10.6 \times 0.15$ , Fig. 4(a)  $\beta = 7.4 \times 11.3 \times 0.15$ , Fig. 4(b) and 4(c)  $\alpha = 4.9 \times 4.4 \times 1.0$ , Fig. 4(b) and 4(c)  $\beta = 4.4 \times 4.9 \times 1.0$ . Three different materials have been used in this work: NinjaFlex (used unless stated otherwise), which is a formulated thermoplastic polyurethane material with super elastic properties; hard PLA; and FlexPLA, which is more flexible compared to hard PLA. More details on material properties at <http://vexmatech.com/>. Hard PLA is brittle at room temperature, which allows us to study the fracture of the auxetic structures.

## C. Tension and compression

Prepared samples are put under tensile and compression load by an Instron Electropuls E1000 testing machine, which applied a constant strain-rate of  $0.02 \text{ mm sec}^{-1}$  up to 1 kN force limitation. The amount of applied force is recorded every 0.002 seconds and a gray scale camera (Dalsa Genie HM1024) records pictures every second.

## D. Simulation

The lattice is simulated as a simple elastic network using molecular dynamics, with LAMMPS. A harmonic spring is placed between every vertex, as well as a harmonic angular spring between each of these bonds. Tension was applied by fixing the velocity of the top row of vertices. The fitting parameters were the bond ( $k_B$ ) and angle ( $k_A$ ) spring constants, however only their ratio affected the results, providing that the imposed tension velocity was sufficiently slow. The results presented in Fig. 2(a) and 2(c) use  $k_A/k_B = 0.122$ .

To allow the links of the lattice to bend, to fit the behavior of the experimental sample after the auxetic regime, each link is replaced by 4 equal length bonds in series. An angular spring is added at every vertex along the link, which is straight in equilibrium and has a spring stiffness ( $k_L$ ). In Fig. 2(d) the parameters used are  $k_A/k_B = 0.2$  and  $k_L/k_B = 0.005$ .

## III. RESULTS

### A. Elastic properties

We find that for all values of disorder, the Poisson's ratio remains negative under both tension, Fig. 2(a), and compression, Fig. 2(b). Generally the Poisson's ratio increases with disorder (the structure becomes less auxetic). Under tension, for both the simulation and experimental results, this trend becomes more evident for values of  $d$  greater than 0.7. In each case we also plot the Young's modulus, and find that it varies more with disorder under compression than tension.

The trend in Poisson's ratio with disorder is further explored in simulations of larger lattice sizes (using a  $16 \times 16$  cell lattice rather than the  $9 \times 9$  cell experimental size), with 100 realizations of each value of disorder, plotted in Fig. 2(c). We have seen that there is no variation in Poisson's ratio with sample size (3), thus these results can be compared with the experiment. As another measure of the amount of disorder in a lattice, we classified the number of topological defects. This is plotted as the percentage of hexagons. The point where the Poisson's ratio starts to change more rapidly with disorder corresponds to the point where the topology of the geometry

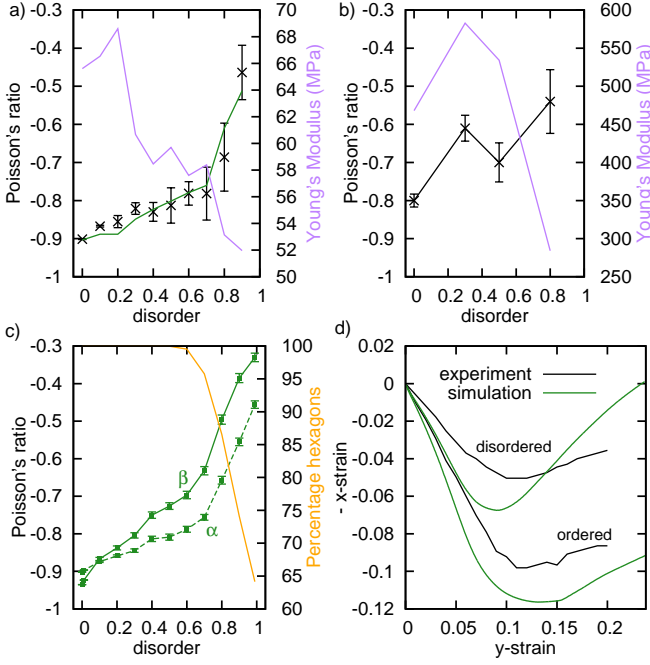


Figure 2. **Behavior of flexible re-entrant lattices.** a) **Tension.** Experimental Poisson's ratio in black, with simulation values from the same geometries in green. Tension applied in the  $\beta$  direction. Error bars are from variation in the middle rows of the lattice. The experimental Young's modulus is in purple. b) **Compression.** Poisson's ratio for samples under compression in the  $\beta$  direction in black, Young's modulus shown in purple. c) **Large scale simulations of tension.** Poisson's ratio in green with error bars from 100 repetitions, the average percentage of hexagons in yellow. d) **Details of tension.**  $y$ -strain vs  $-x$ -strain for tension in the  $\beta$  direction. In the disordered sample  $d = 0.9$ .

first changes. Small changes in topology result in large changes in the Poisson's ratio.

The Poisson's ratio is the slope of the  $y$ -strain vs.  $-x$ -strain plot, included as Fig. 2(d) for the lattice under tension. We see that for both the disordered and ordered samples, the lattice is only auxetic initially, before a turning point where the Poisson's ratio becomes positive. The disordered sample is auxetic over a smaller range of strains than the ordered structure. To fit this behavior after the auxetic regime in simulation, we had to allow each rib to bend in-plane. This was achieved by adding more nodes to the elastic network (see methods).

### B. Out of plane buckling

The change to non-auxetic behavior during the tension experiments, plotted for two materials in Fig. 4(a) and 3(b), corresponds to the sample buckling out of the plane, pictured in Fig. 4(c). The point where the buckling starts, and so the range of strains over which the sample is auxetic, is highly dependent on the sample thickness.

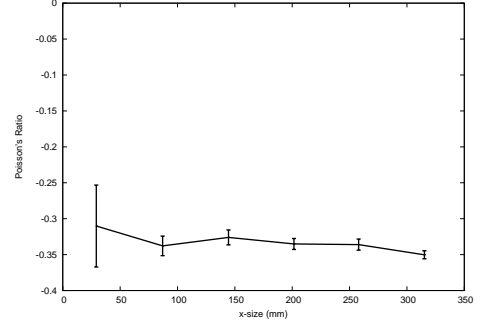


Figure 3. **Size dependence of disordered re-entrant lattice.** Poisson's ratio measured in simulations of different lattice sizes. Error bars are the standard deviation from 100 realizations of the disorder. All geometries were disordered with  $d = 0.9$ .

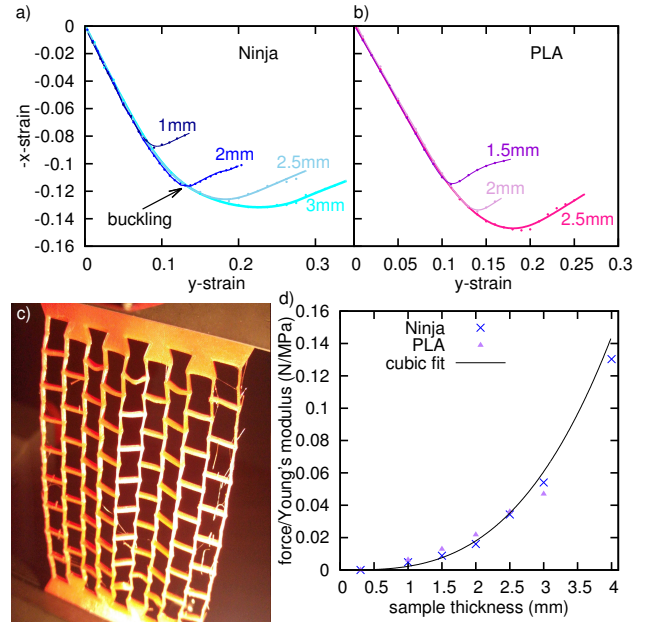


Figure 4. **Out of plane buckling under tension.** a) Onset of buckling in the  $y$ -strain vs  $-x$ -strain plots, shown for NinjaFlex samples of different thicknesses. b) FlexPLA samples. c) Experimental sample buckles in a wave pattern, with alternating vertical strips in front of and behind the clamped plane. d) Sample thickness plotted against normalized force at the onset of buckling. A cubic fit is plotted in black.

While buckling in systems under tension is rare, it has been observed when the boundaries are clamped, and its onset depends cubically on sample thickness and linearly on the Young's modulus [28]. We observed the same relationship, plotted in Fig. 4(d).

### C. Lattice failure

We investigate the fracture properties of the auxetic lattices with samples printed in hard PLA (see methods),

a more brittle filament. The mechanism for fracture under tension in the two orientations we have considered is vastly different. Fig. 5(a) shows that the fracture in the  $\beta$  direction is sudden and brittle, while in the  $\alpha$  direction ductile behavior is observed, with the formation of a shear band preceding the fracture of the structure. This can be seen clearly in the supplementary videos. Details of the fracture process are presented in Fig. 6. In the  $\alpha$  direction, the ultimate tensile strength (UTS) shows no dependence on the disorder in the lattice until high values, when the topology of the lattice includes defects. The sample is stronger in the  $\beta$  direction, though the UTS decreases with added disorder, approaching the UTS for the  $\alpha$  direction. Under compression, the force-strain plots in Fig. 5(b) show little dependence on the orientation of the lattice or the amount of disorder. All samples exhibit brittle failure, though the disordered structures fail at slightly smaller strains. In all cases fracture occurs while the sample is in the auxetic regime.

In flexible samples, compression does not result in fracture. The force strain curves of flexible samples, shown in Fig. 5(c), illustrate the difference in the response to compression in each orientation. In the  $\alpha$  direction each peak in the plot corresponds to one row of the re-entrant lattice collapsing, exhibiting similar low shear resistance to the tensile experiment. In contrast, the force strain plot in the  $\beta$  orientation is simpler, with the lattice first compressing symmetrically in the linear regime, before long wavelength in-plane buckling is observed.

#### IV. SUMMARY AND CONCLUSIONS

In this work we have thoroughly investigated the re-entrant hexagonal lattice structure. We have measured its elastic behavior under both compression and tension, and where applicable we have confirmed the effects of the geometry with computer simulation. Unlike previous studies of disordered re-entrant hexagonal lattices [16, 17], we find the addition of disorder does not have a large effect on the Poisson's ratio, unless the topology is modified. Likewise, the disorder does not have a significant effect on the mechanism of fracture, and is much less important than the orientation of applied tension, where we see brittle failure in one direction but ductile failure in the other, and a notable difference in the sensitivity to disorder.

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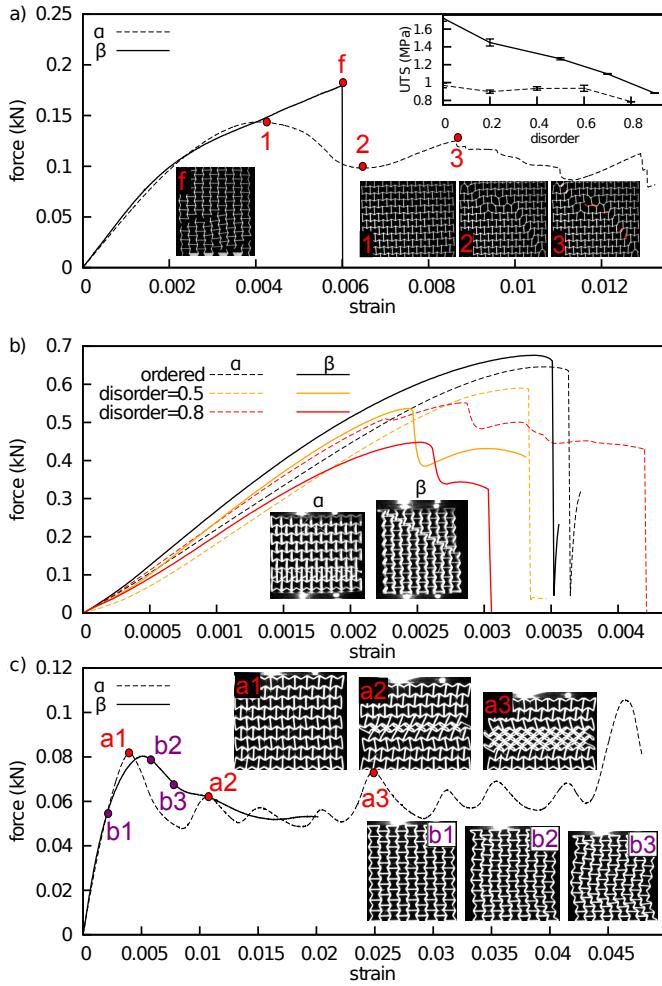


Figure 5. a) **Brittle lattice under tension.** Force strain plot of ordered lattices. The structure of the  $\beta$  lattice after the sudden fracture is labelled as f, and the structure of the lattice at the numbered points on the  $\alpha$  curve are shown. The colors in the last image represent the order of fracture (dark red: first, light pink: last). Insert: the UTS is plotted as a function of disorder. Error bars are calculated from repeats with different geometries. b) **Brittle lattice under compression.** Photographs show fracture of ordered lattices. c) **Flexible ordered lattice under compression.** Labelled photographs throughout the compression are included for both orientations.

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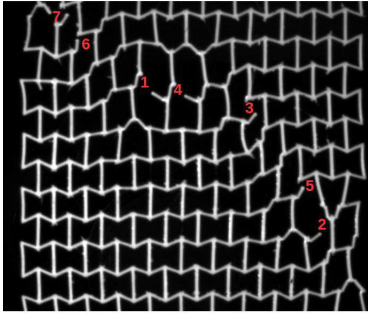


Figure 6. **Fracture of ordered lattice.** Final structure of an ordered experimental sample after tension has been applied in the  $\alpha$  direction. The broken bonds are numbered by the order in which they broke.